CCI Land Surface Temperature

Re-gridding and Sub-setting ATBD

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Applicable Documents

AD-1	Dodd, E., Jimenez, C. and Ghent, D. (2021) Product Specification Document. V2.00
AD-2	Bulgin, C. and Ghent, D. (2021) End-To-End ECV Uncertainty Budget. V2.00
AD-3	Alfred, F., Good, E., Bulgin, C., Rayner, N., Ghent, D. (2021) User Requirements Document (URD). V2.10

1. Background

1.1. Objectives

The Re-gridding and sub-setting tool has the following objectives:

- To satisfy a major new user need [AD-3], specifically related to the requirement for multiple spatial resolutions for local to global users from 0.01˚ to 10.0˚, which would therefore help us to expand the user group substantially
- Starting with the input L3C 0.01° products the tool correctly propagates the LST, corresponding uncertainties, and any ancillary data to coarser spatial resolutions

The expected impact is provision of LST ECV products to all levels of users from local to global to increase downstream exploitation of the LST_cci data archive.

1.2. Rationale

Following the LST_cci User Workshop 24-26 June 2020 it was evident the delivery of LST ECV products at customised spatial resolutions is a major barrier to further uptake of the products by the user community. The standard Level-3C gridded product is produced as daily (day and night) 0.01° equal-angle global datafiles. This meets the general consensus from the users is that highest resolution gridded data possible should be provided rather than prescribing a set resolution. However, for many climate modellers for example this resolution is too fine, and they require an easy and robust method of re-gridding the data to customised spatial resolution to better serve the different model resolutions.

Producing data at such high spatial resolution creates a challenge for many users both in managing the large data volumes and applying knowledge on how best to work with it. While it is possible to provide numerical recipes for correct propagation of the data including the uncertainties, these are non-trivial and still relies on the end user having the capacity to manipulate such big data. Instead, being able to do this via easy-to-use software, which correctly aggregates the data and propagates the uncertainties, interacting directly with the data on a system such as JASMIN would remove these barriers. The aggregated data could then be subset with only the user required data then downloaded.

2. ESA LST_cci Data

2.1. ESA LST_cci Data Products

The re-gridding and sub-setting tool will support all LST_cci data products delivered to the ESA Open Data Portal. This includes IR and MW LST ECV Products from geostationary (GEO) and low earth orbit (LEO) satellites containing information from single satellite instruments, a series of instruments, or a group of merged instruments. Currently, **LST single-sensor ECV Products and CDRs** are available from the following sensors and data levels at the specified spatial resolutions:

Table 1: LST_cci products available on the CCI Open Data Portal

Sensor	Data Level	Spatial resolution	Temporal resolution	Retrieval Algorithm
ATSR-2	L3C	0.01°	Daily (day and night)	UOL
AATSR	L3C	0.01°	Daily (day and night)	UOL
MODIS Terra	L3C	0.01°	Daily (day and night)	GSW
MODIS Aqua	L3C	0.01°	Daily (day and night)	GSW
Sentinel-3x SLSTR	L3C	0.01°	Daily (day and night)	UOL
Metop-A AVHRR	L3C	0.01°	Daily (day and night)	GSW
NOAA-xx AVHRR	L3C	0.05°	Daily (day and night)	GSW
MSGx SEVIRI	L3U	0.05°	Hourly	GSW
GOESx Imager	L3U	0.05°	3-hourly	SMW
GOES16 ABI	L3U	0.05°	Hourly	GSW
MTSATx JAMI	L3U	0.05°	3-hourly	SMW
Himawari-8 AHI	L3U	0.05°	Hourly	GSW
SSM/I and SSMIS CDR	L3C	0.25°	Daily (ascending and descending)	NNEA
AMSR-E and AMSR2 CDR	L3C	0.125°	Daily (ascending and descending)	NNEA
Merged GEO-LEO IR LST CDR	L3S	0.05°	3-hourly	GSW
ATSR-SLSTR LST CDR	L3S	0.01°	Daily (day and night)	UOL

2.2. ESA LST_cci L3C Data Format

LST_cci data products made available on the ESA CCI Open Data Portal (ODP) come in three different data levels:

- L3U products, which are disseminated by the ESA LST_cci project for geostationary satellite products only, are L2 (swath) data from a single instrument that are mapped onto a space-time grid but do not combine data from different orbits.
- L3C products are collated products containing multiple L2P swaths from a single instrument that have been combined and mapped onto a space-time grid. Data are delivered in two separate files for each temporal resolution (either "day" and "night", "ascending" and "descending", or "daily" or "monthly" depending on the product).
- L3S products are L2 data from multiple instruments combined in a space-time grid. LST CDR products are generally L3S products.

The dimensions of the data in each file are described in Table 2. The data are stored in variables in the NetCDF file with the names given in Table 3. More details are available in [AD-1].

Table 2: The dimensions of the data in a L3U / L3C / L3S file.

Dimension Name	Description
lat	These are the dimensions of the regular latitude-longitude grid on which the data are stored.
lon	
time	This is always 1 for L3U / L3C / L3S data.
channel	Channel dimension for the channel variable, which gives the channel wavelengths used to derive LST data.
length_scale	Uncertainty correlation length scale

*Table 3: Data arrays stored in L3U / L3C / L3S data files. The dimensions of the arrays are given in parenthesis after the name of the variable in the NetCDF file. The dimensions are defined in Table 2. Note, not every variable will be available in each of the LST_cci products. The variables marked with a * are common to all outputs.*

Category	Name of data (size of array)	Description
Coordinates	*time (time)	Coordinate variable; time of each temporal point of the data arrays; the start time of the orbit, granule or disk.
	*dtime (time x lat x lon)	Time differences of LST retrievals from the base time in the "time" coordinate variable
	*lat (lat)	Coordinate variable; central latitude of each spatial point of the data arrays
	*lon (lon)	Coordinate variable; central longitude of each spatial point of the data arrays
	channel (channel)	Coordinate variable; sensor channel information
Geophysical variables	*lst (time x lat x lon)	Best available LST retrievals; fill values to be provided where there is ocean (ice free or ice covered) or cloud.
	lcc (time x lat x lon)	Land cover classification of the pixel (biome).
	lst_time_correction (time x lat x lon)	Time correction offset for MW CDR provided as a separate field for users to apply to LST.
Uncertainty information – total uncertainty	*lst_uncertainty (time x lat x lon)	Per pixel total uncertainty of the LST retrieval. Calculated by adding the individual uncertainty components ("lst_unc_ran", "lst_unc_loc_atm", "lst_unc_loc_sfc", "lst_unc_sys") in quadrature.
Uncertainty information – individual components	*lst_unc_ran (time x lat x lon)	Random uncertainties, which are uncorrelated (or weakly correlated) on all spatial and temporal scales.
	*lst_unc_loc_atm (time x lat x lon)	Locally correlated atmospheric uncertainties.
	*lst_unc_loc_sfc (time x lat x lon)	Locally correlated biome or surface uncertainties.
	lst_unc_loc_cor (time x lat x lon)	Locally correlated intercalibration / time correction uncertainties for IR CDRs

Category	Name of data (size of array)	Description
	lst_unc_time_correction (time x lat x lon)	Time correction uncertainties for MW CDR
	*lst_unc_sys (length_scale)	Large scale systematic uncertainties, which are correlated on all spatial and temporal scales.
Retrieval information	*satze (time x lat x lon)	The per pixel satellite zenith angle of the observation.
	*sataz (time x lat x lon)	The per pixel satellite azimuth angle of the observation.
	*solze (time x lat x lon)	The per pixel solar zenith angle of the observation.
	*solaz (time x lat x lon)	The per pixel solar azimuth angle of the observation.
	n (time x lat x lon)	Number of L2P pixels flagged as clear-sky which have contributed to the L3 pixel for IR products, or number of L2P pixels which have contributed to the L3 pixel for MW products.
Quality information	*qual_flag (time x lat x lon)	Per pixel quality flags for each LST retrieval.

As noted by the _FillValue attribute, the number inserted into the data array where no LST was available is -32768. Some tools will identify these automatically.

2.2.1. Using the Uncertainty Estimates

Files contain uncertainties broken down into components from errors that correlate on different spatial and temporal scales:

- Random uncertainties, which are uncorrelated (or weakly correlated) on all spatial and temporal scales, for example random noise in the satellite sensor data.
- Locally correlated atmospheric uncertainties, which is uncertainty assumed to be correlated over distances of 5 km and 5 minutes (related to atmospheric conditions) [AD-2]. So for the purpose of the re-gridding and sub-setting tool it assumed the correlation length scale is 0.05˚.
- Locally correlated biome or emissivity uncertainties, which is assumed to be correlated within the resolution of the CAMEL emissivity dataset (0.05˚) which is used either explicitly for the GSW and SMW algorithms or implicitly for the UOL algorithm [AD-2]. For the data products using the GSW or SMW the emissivity is assumed to be fully correlated within 0.05˚ and 1-month. For the data products using the UOL within a 0.05˚ grid cell all pixels with the same biome are assumed to be fully correlated, and uncollated with pixels of a different biome.
- Large scale systematic uncertainties, which are assumed to be correlated on all spatial and temporal scales (for example related to calibration of the satellite sensor).
- Locally correlated LST correction uncertainties, such as for intercalibration or time corrections, which are assumed to be correlated on specific spatial and temporal scales (for example related to latitude, or land cover). For the purpose of the re-gridding and sub-setting tool it assumed the correlation length scale is 10.0˚ to correspond with the sub-binning by latitude band.
- For each individual LST, the total uncertainty can be obtained by summing each uncertainty component noted above in quadrature (the square root of the sum of squares). The total uncertainty is provided in L3U, L3C and L3S files and is stored in the lst_uncertainty variables contained in the NetCDF files. When re-gridded the lst_uncertainty should be calculated from the individual components rather than propagated directly from the associated input variable. In all cases, correct use of the data requires propagation of the associated uncertainties into the given

application. An exception is the MW CDR, where an uncertainty break down is still not available, and only a total uncertainty figure for the LST, and an uncertainty figure for their time correction are given. For the purpose of the re-gridding and sub-setting it is assumed that these uncertainties are uncorrelated and propagate directly from the input variable.

3. Variable propagation

3.1. Sub-setting

For efficiency, user requested sub-setting will be performed on the input product prior to the re-gridding procedure and will incorporate any pixel which falls within, or overlaps with, the minimum and maximum latitude and longitude selected. For this purpose the calculated corner latitude and longitudes of the grid cells should be used in the selection of the subset grid rather than the centre coordinates which are actual output to the users from the re-gridding and sub-setting process.

3.2. Direct copy

Some variable can be directly copied from the input datafiles to the output datafiles since they are independent of the spatial resolution.

3.3. Remapping

The 1-D latitude and longitude fields need to be remapped onto the new grid with each latitude and longitude representing the centre of the new pixel.

3.4. Non-propagation

Some fields that are not independent of the spatial resolution nevertheless cannot be propagated to a new spatial resolution since they are categorical data they do not translate to new resolutions. These variables will not be written to the new output datafiles.

3.5. Arithmetic sum

Fields that are simple counts of number of pixels, such as “*n*” are propagated by summing all these values from the input datafiles.

3.6. Arithmetic mean

The LST value, for example, within any given grid box is typically calculated as the arithmetic mean of all LST retrievals that fall within the geographical limits of the box (25 for 0.01˚ pixels gridded to 0.05˚).

$$LST_{grid} = \frac{1}{n}\sum_{i=1}^{n} LST_i \qquad \text{(Eq. 1)}$$

Numerous variables follow this arithmetic mean approach for propagating to coarser resolutions.

3.7. Propagating uncorrelated (random) uncertainties

Uncorrelated uncertainties scale by a factor of $1/\sqrt{n}$.

$$u(\langle z\rangle)_i = \frac{1}{\sqrt{n}}.\frac{1}{n}\sum_n u(z)_i \quad \text{(Eq. 2)}$$

Calculation of the arithmetic mean to represent the average gridded LST is based on the assumption that the contributing LST observations representatively sample the LST variation across the grid box. In practice, this is rarely true for infrared LST retrievals as retrievals are only possible under clear-sky conditions. To understand why this is important, it is perhaps useful to consider why uncertainties might vary across a given product. For the LST_cci gridded products, the uncertainty has a dependence on sampling within a grid cell. If lots of observations are available to calculate LST within a grid cell, the uncertainty will typically be lower than when few observations are available. As a consequence, in all cases where cloud obscures some of the grid box, sampling uncertainty is introduced into the averaged LST. This sampling uncertainty is modelled for LST_cci data using the following parameterisation.

$$u(\langle z\rangle)_{samp} = \frac{n_{fill}\sigma_z^2}{n-1} \quad \text{(Eq. 3)}$$

Here, n is the total number of contributing observations to the gridded LST, and n_{fill} is the number of these that are fill values. σ_z^2 is the variance in LST observations across the grid cell. At present an estimate of the variability within a cell has been used, but to improve further would require a significant investigation, which could be initiated in Phase-2.

For the temporal sampling, a similar approach is used. The only difference is that instead of *n* being the total number of contributing sub-grid observations to the gridded LST, *n* is instead the total number of observations over the course of a temporal sampling period that would contribute to the averaged LST for the period. As before n_{fill} is the number of these that are fill values (ie from cloud).

So for example, if the temporal sampling period was 10-days, and there were 10 overpasses which overstruck the grid cell then *n* would be 10. If there were only 3 with valid values then n_{fill} would be 7. The variance σ_z^2 in LST can be quite unstable across a temporal sampling period if the valid observations are spread out or clustered within the period. Therefore, a climatology is used to estimate the variability.

This uncertainty component is uncorrelated between Level 3 grid cells and is therefore added to the uncorrelated (random) uncertainty component as defined in Equation 3.

$$u(\langle z\rangle)_i = \sqrt{u(\langle z\rangle)_i{}^2 + u(\langle z\rangle)_{samp}{}^2} \quad \text{(Eq. 4)}$$

3.8. Propagating fully correlated large-scale uncertainties

The large-scale uncertainty component is fully correlated over the gridded domain. The resultant propagated uncertainty is an average of the input large-scale uncertainties.

$$u(\langle z\rangle)_c = \frac{1}{n}\sum_n u(z)_c \quad \text{(Eq. 5)}$$

3.9. Propagating locally systematic uncertainties

For the locally systematic uncertainties, propagation depends on the respective correlation length scale of the component:

- The correlation length scale of the atmospheric uncertainty component is 5 km (0.05˚) and 5 minutes, so this is assumed fully correlated across a grid cell at this resolution, and uncorrelated outside of this.
- The correlation length scale of the surface uncertainty component for emissivity (GSW and SMW algorithms) is 0.05˚ and monthly, so this is assumed fully correlated across a grid cell at this resolution, and uncorrelated outside of this.
- The correlation length scale of the surface uncertainty component related to the biome (UOL algorithm) is 0.05˚ and monthly, so this is assumed partially correlated across a grid cell at this resolution, and uncorrelated outside of this.
- The correlation length scale of the different correction uncertainty components is 10.0˚, which is the maximum resolution the tool will support, so this is assumed fully correlated across a grid cell at this resolution.

3.9.1. Fully correlated locally systematic propagation

Fully correlated locally systematic uncertainties are propagated according to Equation 6 (which is the same form as Equation 5):

$$u(\langle z\rangle)_{loc} = \frac{1}{n}\sum_{n} u(z)_{loc} \tag{Eq. 6}$$

3.9.2. Uncorrelated locally systematic propagation

Uncorrelated locally systematic uncertainties are propagated according to Equation 7 (which is the same form as Equation 2):

$$u(\langle z\rangle)_{loc} = \frac{1}{\sqrt{n}}.\frac{1}{n}\sum_{n} u(z)_{loc} \tag{Eq. 7}$$

3.9.3. Partially correlated locally systematic propagation

For the surface uncertainties which are based on biome (UOL algorithm), the assumption is made that these are correlated where the biome is consistent, but uncorrelated between biomes. In this case the correlation matrix in the uncertainty calculation would include off-diagonal terms dependent on the underlying biome of the pixels included in the average. This is illustrated for example data in Figure 1.

$$\begin{array}{cccc}
\text{Pixel} & \text{Biome} & & \text{Correlation Matrix} \\
\begin{bmatrix}1\\2\\3\\4\\5\\6\\7\\8\\9\\10\end{bmatrix} &
\begin{bmatrix}A\\A\\D\\C\\C\\B\\A\\D\\A\\C\end{bmatrix} &
\Rightarrow &
\begin{bmatrix}
1&1&0&0&0&0&1&0&1&0\\
1&1&0&0&0&0&1&0&1&0\\
0&0&1&0&0&0&0&1&0&0\\
0&0&0&1&1&0&0&0&0&1\\
0&0&0&1&1&0&0&0&0&1\\
0&0&0&0&0&1&0&0&0&0\\
1&1&0&0&0&0&1&0&1&0\\
0&0&1&0&0&0&0&1&0&0\\
1&1&0&0&0&0&1&0&1&0\\
0&0&0&1&1&0&0&0&0&1
\end{bmatrix}
\end{array}$$

Figure 1: Example correlation matrix for a set of pixels (1-10) with associated biomes (A-D). The correlation matrix has off-diagonal non-zero elements where pixels share the same underlying biome.

We follow the law of propagation of uncertainty for this:

$$u(\langle z \rangle)_{loc}^2 = \sum_{i}^{n} \left(\frac{1}{n}\right)^2 u_i^2(z_i) + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left(\frac{1}{n}\right)\left(\frac{1}{n}\right)u(z_i)u(z_j)r \qquad \text{(Eq. 8)}$$

3.10. Calculating the total uncertainty

The total uncertainty includes components from all the uncorrelated independent effects (*i*), the locally systematic effects (*loc*), and the large-scale common error effects (*c*). The total uncertainty can be estimated as follows, remembering that uncertainties add in quadrature:

$$u(\langle z \rangle)_{total} = \sqrt{u(\langle z \rangle)_i{}^2 + u(\langle z \rangle)_c{}^2 + u(\langle z \rangle)_{loc}{}^2} \qquad \text{(Eq. 9)}$$

3.11. Summary

Some basic guidelines can be followed for the propagation for the different LST_cci products:

- LEO IR 0.01˚
  - If a user selects to re-grid a LST_cci product to 0.05˚ output resolution then the various locally correlated uncertainty components are to be propagated according to respective sections above within their correlation length scales.
  - If a user selects to re-grid a LST_cci product to a resolution coarser than 0.05˚ then **the propagation needs to be performed in two steps**:
    - Propagation to an intermediate 0.05˚ with the various locally correlated uncertainty components being propagated according to respective sections above within their correlation length scales.

    - Propagation of the intermediate 0.05˚ data treating all locally correlated atmospheric and surface components as uncorrelated.
- GEO / Merged IR 0.05˚
  - All re-gridding can be performed in a single step since locally correlated atmospheric and surface components can be treated as uncorrelated beyond their standard resolution.
- MW 0.25˚
  - All re-gridding can be performed in a single step since locally correlated atmospheric and surface components can be treated as uncorrelated beyond their standard resolution.

The following table presents an overview of how each variable is propagated.

Table 4: The variables can be propagated either as i) direct copy as per Section 3.1; ii) remapping as per Section 3.3; iii) via one of the Equations in Sections 3.5 to 3.10; or iv) non-propagation to the new output file.

Category	Variable		Propagation to 0.05˚ Daily Files	Propagation to 0.05˚ Monthly Files	Propagation from 0.05˚ (any temporal period)	(0.05˚ or less) Propagation within 1-month	(0.05˚ or less) Propagation >1-month
Coordinates	Time		Direct copy	Direct copy	Direct copy	Direct copy	Direct copy
	Dtime		Eq. 1	Eq. 1	Eq. 1	Eq. 1	Eq. 1
	Lat		Remapping	Remapping	Remapping	Remapping	Remapping
	Lon		Remapping	Remapping	Remapping	Remapping	Remapping
	Channel		Direct copy	Direct copy	Direct copy	Direct copy	Direct copy
Geophysical variables	Lst		Eq. 1	Eq. 1	Eq. 1	Eq. 1	Eq. 1
	Lcc		Non-propagation	Non-propagation	Non-propagation	Non-propagation	Non-propagation
	lst_time_correction		Eq. 1	Eq. 1	Eq. 1	Eq. 1	Eq. 1
Uncertainty information – total uncertainty	lst_uncertainty	IR	Eq. 9	Eq. 9	Eq. 9	Eq. 9	Eq. 9
		MW	Eq. 2	Eq. 2	Eq. 2	Eq. 2	Eq. 2
Uncertainty information – individual components	lst_unc_ran		Eq. 4	Eq. 4	Eq. 4	Eq. 4	Eq. 4
	lst_unc_loc_atm		Eq. 6	Eq. 7	Eq. 7	Eq. 7	Eq. 7
	lst_unc_loc_sfc	UOL	Eq. 8	Eq. 8	Eq. 7	Eq. 8	Eq. 7
		GSW, SMW, NNEA	Eq. 6	Eq. 6	Eq. 7	Eq. 6	Eq. 7
	lst_unc_loc_cor		Eq. 6	Eq. 6	Eq. 6	Eq. 6	Eq. 6
	lst_unc_time_correction	IR	Eq. 5	Eq. 5	Eq. 5	Eq. 5	Eq. 5
		MW	Eq. 2	Eq. 2	Eq. 2	Eq. 2	Eq. 2
	lst_unc_sys		Eq. 5	Eq. 5	Eq. 5	Eq. 5	Eq. 5

Category	Variable	Propagation to 0.05˚ Daily Files	Propagation to 0.05˚ Monthly Files	Propagation from 0.05˚ (any temporal period)	(0.05˚ or less) Propagation within 1-month	(0.05˚ or less) Propagation >1-month
Retrieval information	Satze	Eq. 1	Eq. 1	Eq. 1	Eq. 1	Eq. 1
	Sataz	Eq. 1	Eq. 1	Eq. 1	Eq. 1	Eq. 1
	Solze	Eq. 1	Eq. 1	Eq. 1	Eq. 1	Eq. 1
	solaz	Eq. 1	Eq. 1	Eq. 1	Eq. 1	Eq. 1
	n	Arithmetic sum	Arithmetic sum	Arithmetic sum	Arithmetic sum	Arithmetic sum
Quality information	qual_flag	Non-propagation	Non-propagation	Non-propagation	Non-propagation	Non-propagation

3.12. Worked Example (inputs)

Here we assume an example of moving from 0.01˚ to 0.05˚ for a Monthly File and for the example for a single pixel, where no time / calibration correction is made.

Table 5: Example values for the 25 0.01˚ pixels within a 0.05˚ cell for LST, lst_unc_ran, lst_unc_loc_atm, lst_unc_loc_sfc (for the GSW algorithm)

LST	301.2	302.12	302.69	303.73	303.52	301.86	Fill	Fill	301.16	301.3	Fill	301.71	300.96	300.67	301.12	303.82	302.79	301.41	301.54	300.9	303.16	303	301.89	301.78	301.12
Ran	3.631	3.783	Fill	2.549	2.154	2.795	Fill	Fill	1.882	Fill	Fill	2.016	Fill	Fill	1.807	Fill	2.053	1.765	Fill	1.772	1.883	1.941	1.68	2.105	2.025
Atm	0.071	0.073	0.075	0.073	0.07	0.079	Fill	Fill	0.075	0.074	Fill	0.074	0.069	0.073	0.074	0.081	0.078	0.068	0.067	0.071	0.083	0.079	0.069	0.067	0.063
Sfc	0.905	0.966	0.979	0.95	0.926	0.889	Fill	Fill	0.877	0.841	Fill	0.853	0.851	0.846	0.813	0.843	0.828	0.821	0.821	0.776	0.816	0.814	0.79	0.769	0.742

The propagation in this example will implement the following equations for each component as detailed in

Table 4:

- lst_unc_ran -> Eq. 4
- lst_unc_loc_sfc -> Eq. 6
- lst_unc_loc_atm -> Eq. 7
- lst_unc_loc_cor -> Not applicable as no time / calibration correction is made in this example
- lst_unc_time_correction -> Not applicable as no time / calibration correction is made in this example
- lst_unc_sys -> Eq. 5

The total number of pixels is 25, which is split as 22 valid pixels and 3 cloudy pixels.

The lst_unc_sys value = 0.03

The extra fill values in lst_unc_ran can be treated as 0 in the equation.

3.13. Worked example (calculations)

3.13.1. lst_unc_ran

Applying Equation 4 to the example can be written such:

$$u(\langle z\rangle)_i = \sqrt{{u(\langle z\rangle)_i}^2 + {u(\langle z\rangle)_{samp}}^2} \qquad \text{(Eq. 4)}$$

This can be substituted by the terms for Equations 2 and 3:

$$u(\langle z\rangle)_i = \sqrt{\frac{\sum_n {u(z)_i}^2}{n^2} + \left(\frac{n_{fill}\sigma_z^2}{n-1}\right)^2}$$

Which can be rewritten in terms of the variable names:

$$lst\_unc\_ran_{0.05} = \sqrt{\frac{\sum_{n_{valid}} {lst\_unc\_ran_{0.01}}_i^{\,2}}{{n_{valid}}^2} + \left(\frac{n_{cloudy} * Variance(LST_{0.01})}{n_{valid} + n_{cloudy} - 1}\right)^2}$$

Filling in with the example values gives:

$$lst\_unc\_ran_{0.05} = \sqrt{\frac{(3.631^2 + 3.783^2 + \cdots + 2.025^2)}{22^2} + \left(\frac{3 * 0.963379}{25-1}\right)^2}$$

Which results in:

$$lst\_unc\_ran_{0.05} = 0.439$$

3.13.2. lst_unc_loc_sfc

Applying Equation 6 to the example can be written such as:

$$u(\langle z\rangle)_{loc} = \frac{1}{n}\sum_n u(z)_{loc} \qquad \text{(Eq. 6)}$$

Which can be rewritten in terms of the variable names:

$$lst\_unc\_loc\_sfc_{0.05} = \sqrt{\frac{\sum_{n_{valid}} {lst\_unc\_loc\_sfc_{0.01}}_i^{\,2}}{n_{valid}}}$$

Filling in with the example values gives:

$$lst\_unc\_loc\_sfc_{0.05} = \sqrt{\frac{(0.905^2 + 0.966^2 + \cdots + 0.742^2)}{22}}$$

Which results in:

$$lst\_unc\_loc\_sfc_{0.05} = 0.853$$

3.13.3. lst_unc_loc_atm

Applying Equation 7 to the example can be written such as:

$$u(\langle z \rangle)_{loc} = \frac{1}{\sqrt{n}} \cdot \frac{1}{n} \sum_{n} u(z)_{loc} \qquad \text{(Eq. 7)}$$

Which can be rewritten in terms of the variable names:

$$lst\_unc\_loc\_atm_{0.05} = \sqrt{\frac{\sum_{n_{valid}} {lst\_unc\_loc\_atm_{0.01}}_i^{\,2}}{{n_{valid}}^2}}$$

Filling in with the example values gives:

$$lst\_unc\_loc\_atm_{0.05} = \sqrt{\frac{(0.071^2 + 0.073^2 + \cdots + 0.063^2)}{22^2}}$$

Which results in:

$$lst\_unc\_loc\_atm_{0.05} = 0.0156$$

3.13.4. lst_unc_sys

Applying Equation 5 to the example can be written such as:

$$u(\langle z \rangle)_{c} = \frac{1}{n} \sum_{n} u(z)_{c} \qquad \text{(Eq. 5)}$$

Which can be rewritten in terms of the variable names:

$$lst\_unc\_sys_{0.05} = \sqrt{\frac{\sum_{n_{valid}} {lst\_unc\_loc\_sys_{0.01}}_i^{\,2}}{n_{valid}}}$$

Filling in with the example values gives:

$$lst\_unc\_sys_{0.05} = \sqrt{\frac{(0.03^2 + 0.03^2 + \cdots + 0.03^2)}{22}}$$

Which results in:

$$lst\_unc\_sys_{0.05} = 0.03$$

3.13.5. lst_uncertainty

Applying Equation 5 to the example can be written such as:

$$u(\langle z\rangle)_{total} = \sqrt{u(\langle z\rangle)_i{}^2 + u(\langle z\rangle)_c{}^2 + u(\langle z\rangle)_{loc}{}^2} \qquad \text{(Eq. 9)}$$

Which can be rewritten in terms of the variable names:

$$lst\_uncertainty_{0.05} = \sqrt{lst\_unc\_ran_{0.05}{}^2 + lst\_unc\_sys_{0.05}{}^2 + lst\_unc\_loc\_atm_{0.05}{}^2 + lst\_unc\_loc\_sfc_{0.05}{}^2}$$

Filling in with the example values gives:

$$lst\_uncertainty_{0.05} = \sqrt{0.439^2 + 0.03^2 + 0.0156^2 + 0.853^2}$$

Which results in:

$$lst\_uncertainty_{0.05} = 0.96$$

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